Derivation of a synthetic beta using the Monte Carlo method as an alternative to the comparable company approach for determining the cost of capital

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Last revision: December 26, 2023

Abstract

The objective of this work is to present an alternative method for approximating the beta coefficient in valuations and investment projects where the risk-return relationship is not observable, making it difficult to obtain the cost of capital for the capital asset pricing model (CAPM).

The proposed methodology implements a simulation model of expected returns based on assumed scenarios for the construction of the free cash flow (FCF). This allows obtaining data to determine the beta coefficient of the project for a segmented investor[2], considering possible idiosyncratic market shocks (positive and negative).

The proposal was applied in a professional valuation of a publicly traded company. The comparison of results confirms the viability of its application in investment projects.

Keywords: *CAPM, beta coefficient, Comparable company beta, twin companies, Private company valuation, Valuation of companies in emerging markets, Project evaluation, return simulation, cost of capital, free cash flow, unsegmented investors. Monte Carlo simulation method.*

JEL Classification codes: G10, G11, G12, G17

This work is available in Spanish in:
https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4755237

Introduction

Generally, the main difficulty in valuing investment projects in emerging countries lies in obtaining the cost of entrepreneurial capital through the application of the Capital Asset Pricing Model (CAPM). This is because the beta coefficient required for its determination cannot be directly observed. This situation generally arises when dealing with a closed-end company, when determining the return on an investment project in a new company, or if the company is already operating, the initiative will be oriented towards a new product line or a combination of the above.

The financial literature indicates that the alternative to avoid this obstacle, is to drawn on to the use of a company that is considered comparable, or the use of ad hoc procedures.

This paper presents an alternative way to obtain the beta coefficient. It is proposed a methodology that addresses two aspects:

a) to avoid using a comparable and,

[1] The views expressed in this work are those of the author and do not necessarily reflect the views of the National Technological University.

[2] This paper assumes that the agent invests all of his/her wealth in a portfolio composed exclusively of project.

b) the possibility of applying the CAPM using expected returns of the project (asset) instead of historical returns, as originally assumed in the model's conception.

This development was carried out based on a valuation performed on Telecom Argentina S.A. in 2008, which is a publicly traded company. This allows for a comparison of the coefficient opportunely applied in its valuation with the one obtained under the proposed methodology.

1. Background

The capital asset pricing model (CAPM), developed by Treynor (1961), Sharpe (1964), Lintner (1965), and Mossin (1966), is the most widely used tool by practitioners and academics to define the required return on an investment. It predicts the theoretical equilibrium price of an asset. It is defined in this postulate:

$$K_e = r_f + \beta_L\left(r_m - r_f\right) \qquad (1)$$

Where k_e is the cost of equity with debt capital, r_f is the risk-free rate, β_L is the levered beta coefficient and r_m is the market return.

In the CAPM, the beta (β) of an asset *j* is defined as its variability. This is the volatility of the market index (r_m) in relation to its average. It quantifies the systematic risk of asset *j*.

$$\beta_j = \frac{\sigma(r_j, r_m)}{\sigma_m^2} = \rho_{(j,m)}\left[\frac{\sigma_j}{\sigma_m}\right] \qquad (2)$$

Where $\sigma(r_j, r_m)$ is the covariance of the return of asset *j* with respect to the return of the market index *m* and σ_m^2 corresponds to the variance of the index. Alternatively, beta is the product of the quotient of the standard deviations of *j* and *m* with the correlation coefficient between asset *j* and the market index *m*. Therefore, beta adjusts the individual unsystematic risk by quantifying only the non-diversifiable aspects.

According to Modigliani and Miller (1958), the value of a levered firm (V_L) is equal to the value of an unlevered firm (V_u) plus the tax benefit (*Ef*) of debt, which is:

$$V_L = V_u + Ef \qquad (3)$$

Hamada (1972) incorporates the Modigliani and Miller (1958) concept of the cost of levered equity capital in the presence of taxes into the CAPM, so the following must be satisfied:

$$r_f + \beta_L\left(r_m - r_f\right) = K_u + \left[\left(K_u - r_f\right)(1-T)\frac{D}{E}\right] \qquad (4)$$

Where k_u is the unlevered cost of equity, *T* is the corporate tax rate, D and E are the market values of debt and equity, respectively.

If we replace k_u with its definition[3] in (4):

$$r_f + \beta_L\left(r_m - r_f\right) = \left[r_f + \beta_u\left(r_m - r_f\right)\right] + \left[\left(\left(r_f + \beta_u\left(r_m - r_f\right)\right) - r_f\right)(1-T)\frac{D}{E}\right] \qquad (5)$$

$$\beta_L\left(r_m - r_f\right) = \beta_u\left(r_m - r_f\right) + \left[\left(\beta_u\left(r_m - r_f\right)\right)(1-T)\frac{D}{E}\right] \qquad (6)$$

Reordering

$$\beta_L = \beta_u\left[1 + \left((1-T)\frac{D}{E}\right)\right] \qquad (7)$$

Solving for

[3] $k_u = r_f + \beta_u(r_m - r_f)$

$$\beta_u = \frac{\beta_L}{\left[1+\left((1-T)\frac{D}{E}\right)\right]} \tag{8}$$

As demonstrated by Fama (1977), the market value of a cash flow is the expected present value discounted at a risk-adjusted rate for each period. He particularly states that, if there is uncertainty about a cash flow to be realized over time, it is not due to the indeterminacy of the future values of "market parameters", such as the market risk premium and risk-free interest rate. A company's beta coefficient is related to the risks of the company's future cash flows. If a project is assumed (where the investor is segmented), its systematic risk is implicit in the variability of the free cash flow.

To carry out the valuation using the CAPM for those assets whose beta coefficient is not observable[4], the specialized literature proposes the option that allows to overcome this inconvenience: to substitute it with one considered comparable.

There are different alternatives for this:

a) using statistical tools[5]
b) through regression of accounting data
c) applying the beta coefficient of a company considered similar or comparable.

In all cases, the value of the coefficient obtained is levered. Therefore, it must be delevered using (8) and re-leveraged[6] to the capital structure being considered, using (7).

The expected return of an asset is the weighted average of all its possible future outcomes. The weights assigned to each outcome, reflecting their relative importance, represent the probability of each event occurring.

This method allows us to determine a parameter that establishes the most probable return of an asset under risky conditions. It achieves this by considering not only all possible random outcomes, but also the degree of probability with which the different scenarios that generate those returns may occur.

The expected return of asset *j* is determined as:

$$E(r_j) = \sum_{n=1}^{t} p_t r_j \tag{9}$$

Where p_t is the probability of occurrence of a given scenario and r_j is the percentage return of asset *j* for that state of nature.

The quantification of the risk of a random variable is represented by the deviations of the different possible outcomes with respect to their expected mean value[7], i.e:

$$\sigma r_j = r_j - \bar{r}_j \tag{10}$$

Where sr_j is the deviation of the return of asset *j* for a given state of nature and $\bar{r}_j$ its average return.

Since deviations can occur both in excess and in deficit, to avoid offsetting results, the variance is taken as a measure of dispersion around the mean value.

$$\sigma^2(r_j) = \left(r_j - \bar{r}_j\right)^2 \tag{11}$$

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[4] Investment projects that are different from and independent of the firm's main activity and/or privately held companies.

[5] Linear regressions to determine the covariance of the asset return with respect to the market return and variance of the market return (see equation 2).

[6] In cases where the discount rate used is the WACC, a calculation circularity arises. See López Dumrauf (2013) pp. 640-646.

[7] Markowitz argues that the sample mean can be used as a proxy for the population mean because it is an unbiased estimator.

2.1 Different methods for obtaining the beta coefficient

The following are the two most commonly used methods for determining the value of companies whose capital is not traded on the stock market.

2.1.1 Comparable beta method

It consists of identifying one or more companies that operate in the stock market whose particularities are reasonably similar to those of the project under study, allowing their replacement with this observable beta coefficient in the determination of the cost of capital. The characteristics that a comparable must have are to belong to the same industrial sector, with a similar cost structure, expenses and operating results before financing. The discrepancy that may arise is with respect to the capital structure (debt/equity ratio). This is solved by deleveraging the comparable and re-leveraging with the corresponding D/E ratio of the asset.

This method assumes that there is a relationship between the return of the twin asset and the market index of a developed country, and that this relationship is the same as that between the expected returns of the local project and the domestic market index.

The use of a comparable company methodology does not take into account specific factors present in the privately held company, which can cause its beta coefficient to vary, such as business model, market position and strategy, among other factors.

There are times when it is not possible to identify a good individual comparable, and it is necessary to use a sector β (an average of the betas of the companies that make up the industry or sector), as indicated by Fuller, R. and Kerr, H. (1981). It is important to note that, in their observations, the line of business operated in the same geographic area.

Erb, C., Harvey, C. and Viskanta, E. (1996) argue that in segmented capital markets, it is not appropriate to use the country beta with respect to the developed country market as a measure of risk. In fact, an incorrect application of this methodology could lead to serious underestimates of the cost of capital in segmented equity markets.

Gray, S. (2008) concluded in his study of a set of Australian energy companies that beta coefficients are generally influenced by "noise", which makes the results obtained statistically unreliable. He observed that estimates of low betas are more likely to be negatively biased and underestimate the true beta.

2.1.2 Accounting beta

It is determined by comparing the company's accounting performance metrics, such as ROE_j with the same ratio for the market (ROE_m).

$$\beta_j = \frac{\sigma\left(ROE_j, ROE_m\right)}{\sigma^2\left(ROE_m\right)} \tag{12}$$

This coefficient obtained by applying equation (12) can be used for any privately held company, although the frequency of data collection is more spaced out than that obtained in the market, given that the time required for accounting registration is longer.

The use of accounting betas is only valid when there is a significant correlation between the company's accounting performance and the market. Its inapplicability is materialized in those cases of projects that do not have previous operating data for its determination.

3. Development

Current knowledge remains inconclusive regarding whether the comparable β coefficient provides an accurate measure for quantifying the systematic risk of a privately held company. However, the study by Fama, E. (1997) on pricing equations suggests that a company's beta coefficient is linked to the risks associated with its future cash flows. This aligns with the current proposal, lending it some validity.

This section presents the empirical case of obtaining the β coefficient for the company Telecom Argentina S. A.

3.1 Method

The procedure proposed in this paper consists of determining the beta coefficient applicable to an investment project[8], obtaining the expected return of the asset by applying the Monte Carlo simulation technique. To do this, the following steps are carried out:

- Assemble the simulation model and determine the parameters of the distribution function of the expected of the project's free cash flow (E(IRR)).
- Calculation of the variance of the expected return of the project, in this case (σ^2_{TE}).
- Calculation of the market's expected return variance (σ^2_m).
- Determination of the covariance between the expected market returns and the expected project returns $\left(\sigma(r_m, r_{TE})\right)$.
- Obtaining the project's systematic risk: $\beta_{TE} = \frac{\sigma(r_m, r_{TE})}{\sigma^2_m}$

The proposal aims to obtain the empirical beta coefficient of the company Telecom Argentina S.A., according to the professional valuation developed by López Dumrauf (2008).

The purpose of using the valuation of the aforementioned company is twofold. First, since it is a publicly traded company, it allows for an objective empirical comparison of the result obtained by applying the proposed method, which can be contrasted with the true beta coefficient calculated from historical market data. Second, to build the simulation of the projected free cash flow (FCF) required by the method (sales growth, evolution of the cost and expense structure, working capital, etc.), the assumptions used in the valuation of Telecom Argentina S.A. have been applied.

3.1.1 Description of the method

To obtain the expected return of an investment project or a privately held company, five states of nature are considered regarding the future prospects of real Gross Domestic Product growth (GDP)[9]. Repetitive simulations of the projected FCF are used, thus obtaining a frequency distribution of the expected return, quantified by its internal rate of return (IRR).

The following describes the different steps to obtain the beta coefficient of an asset not observable in the market. In sections 3.1.2 to 3.1.3, the information and values of the independent input variables for the model construction are presented. In items 3.1.4 to 3.1.9, the partial results for obtaining the simulated beta coefficient are presented.

3.1.2 Definition of the values of the model inputs

Table 1 presents the original growth assumptions used in the professional valuation carried out by López Dumrauf (2013) for the company Telecom S.A., which have been kept constant for the present work.

Table 1 Valuation assumptions applied in the preparation of projected free cash flows

	Projected ratios									
	dec-08	dec-09	dec-10	dec-11	dec-12	dec-13	dec-14	dec-15	dec-16	dec-17
Δ % GDP	8%	5%	4%	4%	3%	3%	3%	3%	3%	3%
Increased sales	16,3%	14,7%	12,8%	11,3%	10,2%	9,2%	8,5%	7,0%	6,0%	5,0%
Cost of sales*	54,0%	52,0%	51,0%	50,0%	50,0%	50,0%	50,0%	50,0%	50,0%	50,0%
Administrative expenses*	3,7%	3,6%	3,5%	3,5%	3,5%	3,5%	3,5%	3,5%	3,5%	3,5%
Marketing expenses*	23,0%	23,0%	23,0%	23,0%	23,0%	23,0%	23,0%	23,0%	23,0%	23,0%

* As percentage of sales

Source: Author's estimates based on valuation data of Telecom S. A. - López Dumrauf (2013)

[8] Or a privately held company.
[9] The probabilities of the expected future macroeconomic states are shown in column P_t of Table 3.

3.1.3 Introduction of risk

Table 8 of the appendix shows the parameterization of the independent input variables of the model for obtaining the simulated FCF. Direct costs, administration and marketing expenses are assumed as a percentage of sales for the period.

For the same, a normal distribution has been adopted, whose mean is the annual variation used in the valuation and its standard deviation has been calculated from the historical data presented in Table 7 of the appendix.

3.1.4 Projected income statement and free cash flow

The income statement and FCF presented in Table 2, have been used in the valuation of Telecom Argentina.

Table 2 Projected income statement and projected free cash flow.

	dec-08	dec-09	dec-10	dec-11	dec-12	dec-13	dec-14	dec-15	dec-16	dec-17
Sales	10.550	12.096	13.642	15.187	16.738	18.283	19.829	21.217	22.490	23.615
Cost of sales	5.697	6.290	6.957	7.594	8.369	9.142	9.915	10.609	11.245	11.807
Gross margin	4.853	5.806	6.684	7.594	8.369	9.142	9.915	10.609	11.245	11.807
Administration expenses	390	435	477	532	586	640	694	743	787	827
Commercialization expenses	2.426	2.782	3.138	3.493	3.850	4.205	4.561	4.880	5.173	5.431
EBIT	2.036	2.588	3.069	3.569	3.933	4.297	4.660	4.986	5.285	5.550

	Per. 0	dec-08	dec-09	dec-10	dec-11	dec-12	dec-13	dec-14	dec-15	dec-16	dec-17
EBIT		2.036	2.588	3.069	3.569	3.933	4.297	4.660	4.986	5.285	5.550
Depreciation + amortization		1.830	1.826	1.881	1.980	2.113	2.270	2.446	2.630	2.816	2.998
EBITDA		3.866	4.415	4.951	5.549	6.046	6.567	7.106	7.617	8.101	8.547
Accounts receivable		143	152	152	152	153	152	152	137	126	111
Inventories		-1	16	18	17	21	21	21	19	17	15
Commercial debts		233	201	220	209	256	254	254	227	209	184
Tax debts		50	46	46	46	47	46	46	42	38	34
Other receivables		-121	31	31	31	31	31	31	28	25	22
Salaries and social sec. contrib.		47	31	31	31	31	31	31	28	25	22
Income tax		713	906	1.074	1.249	1.377	1.504	1.631	1.745	1.850	1.942
Cash flow form operations		3.463	3.587	3.972	4.386	4.798	5.190	5.602	5.985	6.356	6.697
Net change in fixed assets	6.370	1.582	1.814	2.046	2.278	2.511	2.743	2.974	3.183	3.374	3.542
Other long term assets		-5	0	0	0	0	0	0	0	0	0
Cash flow from investments	-6.370	1.577	1.814	2.046	2.278	2.511	2.743	2.974	3.183	3.374	3.542
Free cash flow	-6.370	1.886	1.773	1.926	2.108	2.287	2.447	2.628	2.802	2.982	3.154

Source: Author's estimates based on valuation data from Telecom S. A. - López Dumrauf (2013).
Variation of fixed assets in Per. 0. See table 14.

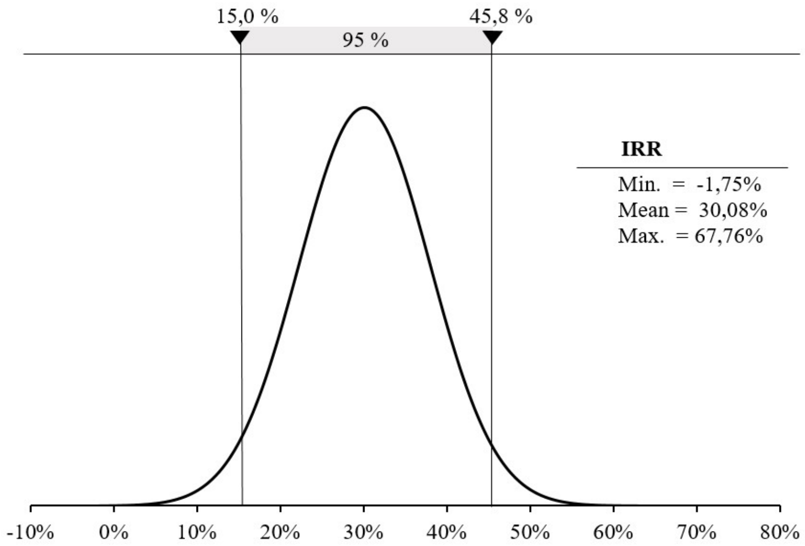
3.1.5 Obtaining the frequency distribution of the company's IRR

Based on the model structure presented in Table 2, the base case free cash flow is simulated[10], using the Monte Carlo method, the distribution of Telecom Argentina's expected return is obtained. The simulated values are presented[11] in Figure 1.

[10] Note: The simulation was performed using the @Risk software with 10 simulations and 200,000 t iterations per simulation.

[11] Note: The data corresponding to the simulated values, maximums, mean, and minimums of the IRR are presented in table 11 of the appendix.

Figure 1 Simulated IRR frequency distribution



Source: Author's estimates.

3.1.6 Calculation of the variance of Telecom Argentina's returns

The table 3 see calculation mechanics for the variance of Telecom Argentina's expected return. This table describes the calculation mechanics for determining the variance of Telecom Argentina's expected return. This is based on the probability of occurrence (P_t), the corresponding state of nature (macroeconomic situation[12]), and the simulated profitability values.

Table 3. Determination of E(r_{TE}) values for a 95% confidence interval and calculation of variance.

States of nature	P_t	r_{TE}	$P_{(t)} r_{TE}$	$P_{(t)} \left(r_{TE} - \overline{r_{TE}} \right)^2$
Highly recessive	2,5%	-1,59%	-0,00040	0,19272
Moderately recessive	15,0%	15,02%	0,02253	0,03687
No change from previous period	60,0%	30,08%	0,18048	0,00012
Moderate recovery	20,0%	45,78%	0,09156	0,03029
Strong recovery	2,5%	67,00%	0,01675	0,21307
	100,0%		$\overline{r_{TE}} = 0,3109$	$\sigma^2_{TE} = 0,473063$

Source: Author's estimates.

Where r_{TE} is the expected return of Telecom Argentina in each scenario; $\overline{r_{TE}}$ is the expected average return of Telecom Argentina, and σ^2_{TE} is its variance.

3.1.7 Calculation of the Merval frequency interval and its variance

To determine the expected annual return of the market index (Merval), with a 95% confidence level, we start from the data presented in Table 4, whose historical values are shown in table 10 of the annex.

[12] Note: The probability of occurrence of each state of nature is based on assumptions about the expected GDP growth for 2008. Please refer to table 1 - professional valuation by López Dumrauf (2013) for the percentage variation of GDP.

Table 4 Determination of values for a 95 % confidence interval for the market index.

Date	Interval
n = 11	Min.= -0,47
σ = 0,3685596	$\overline{X} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} =$ -0,108893
$Z_{\alpha/2}$ = 1,96	Mean = 0,1089113
	$\overline{X} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} =$ 0,3267162
	Max.= 0,7139317

(13)

Source: Author's estimates.

The expected average return of the market index and its variance, are obtained by applying the same method as in 3.1.6, as shown in table 5.

Table 5 Obtaining the variance of MERVAL.

States of nature	P_t	r_m	$P_{(t)}r_m$	$P_{(t)}\left(r_m - \overline{r_m}\right)^2$
Highly recessive	2,5%	-46,91%	-0,011728	0,008691085
Moderately recessive	15,0%	-10,89%	-0,016334	0,007891516
No change from previous period	60,0%	10,89%	0,0653468	8,02445E-05
Moderate recovery	20,0%	32,67%	0,0653432	0,008507009
Strong recovery	2,5%	71,39%	0,0178483	0,008804744
	100,0%		$\overline{r_m}$ = 0,1205	σ_m = 0,03397

Source: Author's estimates.

Where r_m is the expected market return for each state of nature y σ^2_m corresponds to its variance.

3.1.8 Covariance calculation

To determine the covariance between Telecom Argentina's returns and the market index is applied:

$$\sigma_{(TE,m)} = \sum_{n=1}^{t} P_t\left[\left(r_{TE} - \overline{r_{TE}}\right)\left(r_m - \overline{r_m}\right)\right] \tag{14}$$

Table 6 presents the calculation mechanics for obtaining the covariance between the company and the aforementioned index.

Table 6 Covariance calculation

States of nature	P_t	$P_t\left[\left(r_{TE} - \overline{r_{TE}}\right)\left(r_m - \overline{r_m}\right)\right]$
Highly recessive	2,5%	0,00482
Moderately recessive	15,0%	0,00553
No change from previous period	60,0%	0,00007
Moderate recovery	20,0%	0,00606
Strong recovery	2,5%	0,00533
		$\sigma_{(TE,m)}$ = 0,02180

Source: Author's estimates.

3.1.9 Determination of β

From the calculation of the covariance and variance of the market index, using equation (2), we obtain:

$$\beta_U = \frac{0,02180}{0,03397} = 0,64$$

To compare the result obtained with this methodology with the one applied in the valuation[13] , we proceed to re-leverage β_U with the corresponding capital structure, that is, a D/E ratio=0.1869. Using (7)[14], we arrive at:

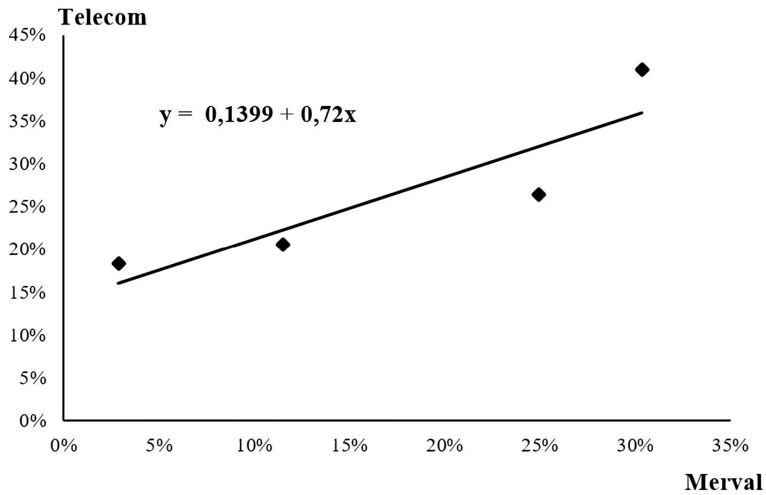
$$\beta_{L'} = 0,72$$

Since the company we are dealing with is publicly traded, obtaining β_L through linear regression using annual data yields a value of 0.72. In figure 2, the equation of the characteristic line of the asset is:

$$E(r_{TE}) = \alpha + \beta_{L'} r_m \quad (15)$$

It can be seen that there is only one factor that affects the systematic risk of the asset, namely the slope of the line or β_L. The data provided by the market for its calculation is presented in Table 10 of the appendix.

Figure 2 Regression line of Telecom Argentina vs. Merval (annual data).



Source: Author's estimates.

Studies, such as those by Altman *et al*. (1974), Sholes and Williams (1977), and others, have shown that the value of the coefficient β can vary depending on the time scale of the returns used in its calculation. The impact on the estimation of β caused by the periodicity of the data used is known as the interval effect. This bias in the estimation of beta has been attributed by Cohen *et al*. (1983) to the lag in the adjustment of asset prices, mainly in small companies.

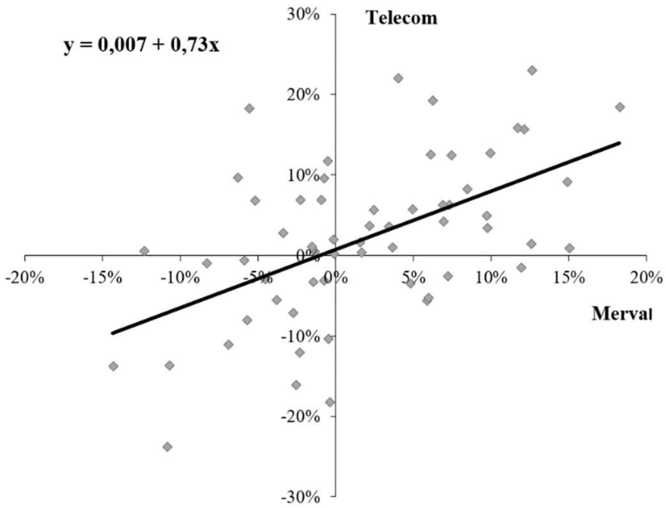
In principle, it would seem preferable to use daily returns over weekly, monthly, or annual returns. However, it can be argued that the effect of new information does not always affect daily or weekly prices, especially in emerging markets, as their adjustment may be delayed. When the time interval is increased, prices contain more information, minimizing the impact caused by delays.

Figure 3 shows the value of β_L = 0.73 obtained by linear regression with the same time period of data, but with monthly frequency (see Table 14).

[13] Note: The leveraged beta (β_L) applied in the valuation performed by López Dumrauf (2013), is 0.73.

[14] Note: Being the tax rate T = 0.35.

Figure 3 Regression line of Telecom Argentina vs Merval (monthly data).



Source: Author`s estimates.

4. Conclusions

The CAPM model is widely used by academics and practitioners for the determination of the value of assets, both financial and real. It is from this model that the importance of the beta coefficient as a risk estimator can be seen.

When using a comparable company when β is not observable, it is assumed that the relationship between the returns of the comparable company and the developed market is similar to the relationship between the returns of the domestic company and the emerging market.

This paper presents an alternative for the application of the CAPM to local companies where their beta coefficient (β) is not observable.

Using simulation tools, information about the company's returns is generated. With the sample of expected returns, it is possible to determine the beta coefficient (β) ratio of a privately held company.

Comparing the $\beta_L = 0.73$ (observable) applied in the valuation of Telecom Argentina S.A., and the $\beta_{L'} = 0.72$ obtained using the proposed methodology, it can be seen that they are very close in value.

In the genesis of the CAPM's operation, β is closely related to the asset's ability to generate future funds, since the model expresses the minimum fair return that should be required.

Beta contributes to the valuation model by quantifying the unique risk that the asset possesses in determining its return. This fact is a condition of the variability of the expected FCF.

It is the opinion of the author of this work that, conceptually, the application of comparable betas to value privately held companies in emerging markets presents inconsistencies. While it may be possible to find a company to use that reasonably meets the requirements stated in 2.2.1, such a company would be operating in a developed nation with a stable economy, a mature legislative and executive branch, and different and stable productive, commercial, labor, and tax regulations compared to most emerging markets.

Legal security, along with legislation, provides an institutional framework for the long-term progress and stability of businesses. These characteristics are not always present in developing markets.

The tools applied by central governments in developed countries to monitor and intervene in the phases of the economic cycle are more effective than identical instruments used in emerging economies.

Therefore, the *management* of "comparable" companies has a different idiosyncrasy in corporate and strategic management, for their development and maximization of the value of the company's shares, compared to companies operating in emerging countries.

Some of the issues mentioned above are only partially reflected by the comparable's beta coefficient. The aforementioned is also not fully captured by the country risk premium.

It is expected that the proposed method will be examined by other authors in different settings than the one tested in this work.

Annex

The values shown in Tables 7 and 14 are historical data that are part of Telecom Argentina's financial statements[15] from 1996-2007.

Table 7. Values, historical sales ratios, mean and variance of model inputs.

	Historical values (in thousands of Pesos)											
	dec-96	dec-97	dec-98	dec-99	dec-00	dec-01	dec-02	dec-03	dec-04	dec-05	dec-06	dec-07
Sales	1.983.274	2.585.000	3.173.000	3.183.000	3.226.000	3.049.000	3.983.000	3.753.000	4.494.000	5.718.000	7.437.000	9.074.000
Cost of sales	1.167.546	1.391.000	1.586.000	1.538.000	1.729.000	1.488.000	2.872.000	2.640.000	2.950.000	3.704.000	4.510.000	4.963.000
Administrative expenses	118.306	178.000	208.000	238.000	235.000	246.000	279.000	222.000	244.000	249.000	272.000	342.000
Marketing expenses	206.515	331.000	562.000	661.000	631.000	903.000	1.034.000	784.000	900.000	1.261.000	1.743.000	2.133.000

	Historical ratios												
	dec-97	dec-98	dec-99	dec-00	dec-01	dec-02	dec-03	dec-04	dec-05	dec-06	dec-07	μ	σ
Δ % GDP	8,1%	3,9%	-3,4%	-0,8%	-4,4%	-10,9%	8,8%	4,7%	8,9%	8,0%	9,0%		
Increased sales	30,3%	22,7%	0,3%	1,4%	-5,5%	30,6%	-5,8%	19,7%	27,2%	30,1%	22,0%	15,74%	14,95%
Cost of sales*	53,8%	50,0%	48,3%	53,6%	48,8%	72,1%	70,3%	65,6%	64,8%	60,6%	54,7%	58,47%	8,26%
Administrative expenses*	6,9%	6,6%	7,5%	7,3%	8,1%	7,0%	5,9%	5,4%	4,4%	3,7%	3,8%	6,04%	1,47%
Marketing expenses*	12,8%	17,7%	20,8%	19,6%	29,6%	26,0%	20,9%	20,0%	22,1%	23,4%	23,5%	20,56%	5,25%

* As percentage of sales

Source: Author’s estimates based on data from López Dumrauf (2013).

[15] Financial statements of Telecom Argentina. Presented in accordance with technical resolutions of the Professional Council of Economic Sciences under the RT89 regulation.

Table 8. Parameters of the model's input distributions (as a percentage of sales).

	Variable	Distribution	μ	σ
dec-08	Increase in sales	Normal	16,3%	14,95%
dec-09	Increase in sales	Normal	14,7%	14,95%
dec-10	Increase in sales	Normal	12,8%	14,95%
dec-11	Increase in sales	Normal	11,3%	14,95%
dec-12	Increase in sales	Normal	10,2%	14,95%
dec-13	Increase in sales	Normal	9,2%	14,95%
dec-14	Increase in sales	Normal	8,5%	14,95%
dec-15	Increase in sales	Normal	7,0%	14,95%
dec-16	Increase in sales	Normal	6,0%	14,95%
dec-17	Increase in sales	Normal	5,0%	14,95%

	Variable	Distribution	μ	σ	
dec-08	Cost of sales	Normal	γ	8,26%	$\gamma = Sales_t$ x 54%
dec-09	Cost of sales	Normal	γ	8,26%	$\gamma = Sales_t$ x 52%
dec-10	Cost of sales	Normal	γ	8,26%	$\gamma = Sales_t$ x 51%
dec-11	Cost of sales	Normal	γ	8,26%	$\gamma = Sales_t$ x 50%
dec-12	Cost of sales	Normal	γ	8,26%	$\gamma = Sales_t$ x 50%
dec-13	Cost of sales	Normal	γ	8,26%	$\gamma = Sales_t$ x 50%
dec-14	Cost of sales	Normal	γ	8,26%	$\gamma = Sales_t$ x 50%
dec-15	Cost of sales	Normal	γ	8,26%	$\gamma = Sales_t$ x 50%
dec-16	Cost of sales	Normal	γ	8,26%	$\gamma = Sales_t$ x 50%
dec-17	Cost of sales	Normal	γ	8,26%	$\gamma = Sales_t$ x 50%

	Variable	Distribution	μ	σ
dec-08	Administrative expenses	Normal	η	1,47% $\eta = Sales_t$ x 3,7%
dec-09	Administrative expenses	Normal	η	1,47% $\eta = Sales_t$ x 3,6%
dec-10	Administrative expenses	Normal	η	1,47% $\eta = Sales_t$ x 3,5%
dec-11	Administrative expenses	Normal	η	1,47% $\iota = Saless_t$ x 3,5%
dec-12	Administrative expenses	Normal	η	1,47% $\eta = Sales_t$ x 3,5%
dec-13	Administrative expenses	Normal	η	1,47% $\eta = Sales_t$ x 3,5%
dec-14	Administrative expenses	Normal	η	1,47% $\eta = Sales_t$ x 3,5%
dec-15	Administrative expenses	Normal	η	1,47% $\eta = Sales_t$ x 3,5%
dec-16	Administrative expenses	Normal	η	1,47% $\eta = Sales_t$ x 3,5%
dec-17	Administrative expenses	Normal	η	1,47% $\eta = Sales_t$ x 3,5%

	Variable	Distribution	μ	σ
dec-08	Marketing expenses	Normal	φ	5,25% $\varphi = Sales_t$ x 23%
dec-09	Marketing expenses	Normal	φ	5,25% $\varphi = Sales_t$ x 23%
dec-10	Marketing expenses	Normal	φ	5,25% $\varphi = Sales_t$ x 23%
dec-11	Marketing expenses	Normal	φ	5,25% $\varphi = Sales_t$ x 23%
dec-12	Marketing expenses	Normal	φ	5,25% $\varphi = Sales_t$ x 23%
dec-13	Marketing expenses	Normal	φ	5,25% $\varphi = Sales_t$ x 23%
dec-14	Marketing expenses	Normal	φ	5,25% $\varphi = Sales_t$ x 23%
dec-15	Marketing expenses	Normal	φ	5,25% $\varphi = Sales_t$ x 23%
dec-16	Marketing expenses	Normal	φ	5,25% $\varphi = Sales_t$ x 23%
dec-17	Marketing expenses	Normal	φ	5,25% $\varphi = Sales_t$ x 23%

Source: Author’s estimates.

Table 9. Historical market index performance. Annual price and yield data.

Merval		
Date	Close	Δ%
dec-96	649,37	
dec-97	687,50	5,71%
dec-98	430,06	-46,91%
dec-99	550,47	24,68%
dec-00	416,77	-27,82%
dec-01	295,39	-34,42%

Merval		
Date	Close	Δ%
dec-02	524,95	57,50%
dec-03	1.071,95	71,39%
dec-04	1.375,37	24,92%
dec-05	1.543,31	11,52%
dec-06	2.090,46	30,35%
dec-07	2.151,73	2,89%

Source: Author’s estimates.

Table 10. Annual Price and Return Data for Merval and Telecom Argentina

Date	Merval	Telecom	Δ % Merval	Δ % Telecom
dec-03	1.071,95	4,94		
dec-04	1.375,37	6,43	24,92%	26,36%
dec-05	1.543,31	7,90	11,52%	20,59%
dec-06	2.090,46	11,90	30,35%	40,97%
dec-07	2.151,73	14,30	2,89%	18,37%

Source: Author’s estimates.

Table 11 Simulated FCF values.

			Simulation										
Free cash Flow	Iterations	Value	N° 1	N° 2	N° 3	N° 4	N° 5	N° 6	N° 7	N° 8	N° 9	N° 10	Avg
dec-08	200.000	Min	488	434	368	504	522	489	457	505	347	468	458
		Avg	1.886	1.886	1.886	1.886	1.886	1.886	1.886	1.886	1.886	1.886	**1.886**
		Max	3.234	3.239	3.235	3.346	3.257	3.332	3.231	3.301	3.395	3.273	3.284
dec-09	200.000	Min	-138	114	72	143	137	-149	151	-13	150	128	60
		Avg	1.773	1.773	1.772	1.773	1.772	1.773	1.773	1.773	1.773	1.773	**1.773**
		Max	4.133	4.403	4.235	4.332	4.049	4.196	4.191	4.193	3.961	3.933	4.163
dec-10	200.000	Min	15	-65	-142	10	-163	-175	-62	-25	-81	-179	-87
		Avg	1.926	1.926	1.926	1.926	1.926	1.926	1.926	1.926	1.926	1.926	**1.926**
		Max	5.367	5.077	5.303	5.800	5.537	5.730	5.455	5.558	5.639	5.693	5.516
dec-11	200.000	Min	-72	-46	-109	-191	-111	-47	-96	-96	-141	-101	-101
		Avg	2.108	2.107	2.108	2.108	2.108	2.108	2.108	2.107	2.108	2.108	**2.108**
		Max	7.133	6.895	8.448	7.505	7.428	7.507	7.819	6.590	6.621	7.397	7.334
dec-12	200.000	Min	-213	-196	-272	-230	-246	-220	-147	-312	-294	-177	-231
		Avg	2.288	2.287	2.287	2.288	2.287	2.288	2.288	2.287	2.287	2.288	**2.287**
		Max	8.719	8.684	8.591	9.455	8.473	8.698	10.096	9.572	9.877	10.791	9.296
dec-13	200.000	Min	-228	-304	-347	-492	-334	-327	-400	-699	-266	-284	-368
		Avg	2.447	2.448	2.447	2.447	2.448	2.447	2.448	2.447	2.448	2.447	**2.447**
		Max	10.927	11.090	11.256	10.487	10.484	11.361	11.079	10.783	11.784	12.140	11.139
dec-14	200.000	Min	-589	-341	-409	-302	-350	-316	-461	-600	-299	-388	-405
		Avg	2.629	2.628	2.628	2.627	2.627	2.628	2.629	2.627	2.628	2.628	**2.628**
		Max	13.582	14.150	12.870	15.175	13.603	13.300	13.434	12.994	13.594	12.409	13.511
dec-15	200.000	Min	-438	-453	-389	-688	-373	-363	-355	-396	-446	-458	-436
		Avg	2.802	2.802	2.802	2.802	2.802	2.802	2.803	2.801	2.802	2.802	**2.802**
		Max	17.143	15.756	18.331	15.240	14.282	16.332	15.385	15.350	16.998	14.718	15.954
dec-16	200.000	Min	149	693	-564	907	514	662	561	643	957	723	525
		Avg	2.982	2.982	2.982	2.982	2.982	2.982	2.982	2.982	2.982	2.982	**2.982**
		Max	4.884	4.717	4.592	4.886	4.611	4.619	4.704	4.456	4.974	4.504	4.695
dec-17	200.000	Min	44	4	4	19	-29	59	-5	91	95	33	32
		Avg	3.155	3.155	3.154	3.155	3.154	3.155	3.155	3.152	3.154	3.154	**3.154**
		Max	17.792	17.655	16.186	16.072	15.523	17.398	17.178	16.869	19.072	16.421	17.017

Source: Author's estimates.

Table 12. IRR values obtained in the simulations

Output variable	Number of Iterations	Simulation	Simulated values		
			Minimum	Mean	Maximun
IRR	200.000	1	-0,70%	30,08%	71,69%
IRR	200.000	2	-0,17%	30,08%	65,14%
IRR	200.000	3	-4,17%	30,08%	67,38%
IRR	200.000	4	-0,22%	30,08%	70,47%
IRR	200.000	5	-0,91%	30,08%	65,79%
IRR	200.000	6	-2,36%	30,08%	67,74%
IRR	200.000	7	-3,05%	30,08%	69,42%
IRR	200.000	8	0,01%	30,08%	65,15%
IRR	200.000	9	-2,72%	30,08%	67,92%
IRR	200.000	10	-3,21%	30,08%	66,90%
		Mean	-1,75%	30,08%	67,76%

Source: Author's estimates.

Table 13. Monthly price and yield data for the market index and Telecom.

Date	Merval	Telecom	Date	Merval	Telecom
may-03	678,31	3,20	nov-05	1.554,67	8,15
jun-03	765,61	3,74	dec-05	1.543,31	7,90
jul-03	755,34	3,76	jan-06	1.793,97	7,97
aug-03	713,33	3,47	feb-06	1.714,05	7,74
sep-03	827,69	3,80	mar-06	1.800,58	8,20
oct-03	929,89	4,45	apr-06	1.908,61	7,75
nov-03	996,56	4,64	may-06	1.653,72	6,75
dec-03	1.071,95	4,94	jun-06	1.711,09	7,00
jan-04	1.140,81	5,99	jul-06	1.701,58	7,87
feb-04	1.183,14	6,05	aug-06	1.662,84	8,43
mar-04	1.201,66	6,15	sep-06	1.637,27	8,52
apr-03	1.077,93	4,85	oct-06	1.781,68	9,25
may-04	952,62	4,88	nov-06	1.967,02	10,50
jun-04	945,45	5,37	dec-0	2.090,46	11,90
jul-04	966,10	5,57	jan-07	2.070,64	12,75
aug-04	952,14	5,39	feb-07	2.067,64	13,00
sep-04	1.142,50	6,48	mar-07	2.102,78	13,05
oct-04	1.287,14	6,38	apr-07	2.154,55	13,80
nov-04	1.213,09	6,34	may-07	2.243,03	17,20
dec-04	1.375,37	6,43	jun-07	2.190,87	15,25
jan-05	1.373,79	6,44	jul-07	2.180,25	13,75
feb-05	1.558,62	8,11	aug-07	2.062,08	16,50
mar-05	1.400,42	7,07	sep-07	2.187,97	15,65
apr-05	1.348,35	6,69	oct-07	2.351,44	15,25
may-05	1.485,55	7,03	nov-07	2.207,16	16,80
jun-05	1.367,41	6,96	dec-07	2.151,73	14,30
jul-05	1.507,59	7,20	jan-08	2.007,27	12,80
aug-05	1.581,65	6,95	feb-08	2.162,20	14,50
sep-05	1.694,83	7,40	mar-08	2.103,72	13,50
oct-05	1.608,86	7,92	apr-08	2.095,53	11,25

Date	Δ% Merval	Δ% TECO	Date	Δ% Merval	Δ% TECO
jun-03	12,1%	15,6%	dec-05	-0,7%	-3,1%
jul-03	-1,4%	0,5%	jan-06	15,1%	0,9%
aug-03	-5,7%	-8,0%	feb-06	-4,6%	-2,9%
sep-03	14,9%	9,1%	mar-06	4,9%	5,8%
oct-03	11,6%	15,8%	apr-06	5,8%	-5,6%
nov-03	6,9%	4,2%	may-06	-14,3%	-13,8%
dec-03	7,3%	6,3%	jun-06	3,4%	3,6%
jan-04	6,2%	19,3%	jul-06	-0,6%	11,7%
feb-04	3,6%	1,0%	aug-06	-2,3%	6,9%
mar-04	1,6%	1,6%	sep-06	-1,5%	1,1%
apr-03	-10,9%	-23,7%	oct-06	8,5%	8,2%
may-04	-12,4%	0,6%	nov-06	9,9%	12,7%
jun-04	-0,8%	9,6%	dec-0	6,1%	12,5%
jul-04	2,2%	3,7%	jan-07	-1,0%	6,9%
aug-04	-1,5%	-3,3%	feb-07	-0,1%	1,9%
sep-04	18,2%	18,4%	mar-07	1,7%	0,4%
oct-04	11,9%	-1,6%	apr-07	2,4%	5,6%
nov-04	-5,9%	-0,6%	may-07	4,0%	22,0%
dec-04	12,6%	1,4%	jun-07	-2,4%	-12,0%
jan-05	-0,1%	0,2%	jul-07	-0,5%	-10,4%
feb-05	12,6%	23,1%	aug-07	-5,6%	18,2%
mar-05	-10,7%	-13,7%	sep-07	5,9%	-5,3%
apr-05	-3,8%	-5,5%	oct-07	7,2%	-2,6%
may-05	9,7%	5,0%	nov-07	-6,3%	9,7%
jun-05	-8,3%	-1,0%	dec-07	-2,5%	-16,1%
jul-05	9,8%	3,4%	jan-08	-6,9%	-11,1%
aug-05	4,8%	-3,5%	feb-08	7,4%	12,5%
sep-05	6,9%	6,3%	mar-08	-2,7%	-7,1%
oct-05	-5,2%	6,8%	apr-08	-0,4%	-18,2%
nov-05	-3,4%	2,9%			

Source: Author's estimates.

Table 14. ECCC as of 12/31/07. Variation in property, plant and equipment. Determination of the theoretical investment amount.

	dic-07		dic-07
Cash and banks	45	Commercial debt	1.640
Transitional investmen	947	Salaries and social security charges	164
Accounts receivable	898	Tax debts	266
Inventories	157	Short-term financial debt	1.474
Other assets short term	5	other short term liabilities	50
Other credits	332	Provisions	49
Total current assets	**2.384**	**Total current liabilities**	3.643
Property and equipment	5.738	Long term financial debts	1.724
Long term investments	2	Long term provisions	243
Intangibles	760	Long term social and tax debts	332
Other accounts receivable long term	282	Other long term liabilities	120
Other long term assets	5	**Total non current liabilities**	2.419
Total non current assets	**6.787**	**Pasivos totales**	**6.062**
Total assets	**9.171**	**Total liabilities**	**3.109**
		Total Liabilities + Equity	**9.171**

I) Current operating assets = 1.437
II) Current operating liabilities = 1.854
III) Operating working capital = -417
VI) Non current assets = 6.787

Theoretical investment value (III+ IV) = -6.370

Source: Author's estimates.

References

Altam, E., Jacquillat, B. and Levasseur, M. (1974). Comparative analysis of risk measures: France and the United States. *The Journal of Finance, 29*(5), 1945-1511.

Alvarez, A., Graffi, H. and Messuti, D. (1992). *Selecciòn de inversiones. Introducción a la teoría de carteras (Portfolio theory).* Buenos Aires: Ediciones Macchi.

Cohen, K., Hawawini, G., Maier, S., Shawartz, R. and Whitcomb, D. (1983). *Friction in the trading process and the estimation of systematic risk.* Journal of Financial Economics, *60*(3), 425-448.

Fama, E. (1977). *Risk-Adjusted Discount Rates And Capital Budgeting Under Uncertainty.* Journal of financial Economics, *5*, 3-24.

Godfrey, S and Espinosa, R. (1996). *A Practical Approach to Calculating Costs of Equity for Investments in Emerging Markets*. Journal of Applied Corporate Finance*, 9.3*, 80-89.

Hamada, R. (1972). *The effect of the firm's capital structure in the systematic risk.* The Journal of Finance*, 27*, 435-458.

López Dumrauf, G. (2013, September 19). *Consultoría en valuación de empresas: valor justo del holding Sofora Telecomunicaciones S.A.* XXXIII Jornadas Nacionales de Administración Financiera. SADAF. https://economicas.unsa.edu.ar/. https://economicas.unsa.edu.ar/afinan/informacion_general/sadaf/xxxiii_jornadas/xxxiii-j-lopez-dumrauf-valuacion-sofora-present.pdf.

López Dumrauf, G. (2013). *Finanzas corporativas. Un enfoque latinoamericano* (3 ed.). Alfaomega.

Mandagarán Rivas, G. (2023). *Formulación y evaluación de proyectos. Analisis de inversiones en activos fijos en entornos emergentes.* Buenos Aires: Alfaomega, 499-507.

Markowitz, H. (1952). *Portfolio Selection.* The Journal of Finance, *7*(1), 77-91.

Modigliani, F. and Miller, M. (1958). *The cost of capital. Corporation finance and the theory of investment.* American Economic Review, *48*(3), 261-297.

Pereiro, L. and Galli, M. (2000). *La determinación del costo de capital en la valuación de empresas de capital cerrado: una guía práctica*. Argentine Institute of Finance Executives and Torcuato Di Tella University.

Sharpe, W. (1964). *Capital asset prices: A theory of market equilibrium under conditions of risk*. The Journal of Finance*, 19*(3), 425-442.

Sholes, M. and Williams, J. (1977). *Estimating betas from nonsynchronous data*. Journal of Financial Economics*, 5*(3), 309-327.